Thus, modulo a limiting argument (which we give at the end), Theorem 2 will follow from:

<u>Lemma 1</u> Let $0 < p \leq 1$, $\beta > -1$. Let $f \in L^\infty(\mathbb{R}^n)$ and $|\nabla f| \in L^\infty(\mathbb{R}^n)$, and set $u = P_y * f$. If:

$$\int_{\mathbb{R}^{n+1}_+} \left|\frac{\partial u}{\partial y}\right|^p y^\beta \, dx\, dy < \infty$$

then there exist numbers λ_k and (n,p,β)-atoms a_k so that:

$$f = \sum \lambda_k a_k \; \cancel{\text{in } \mathcal{S}'(\mathbb{R}^n)} + (\text{const}) \text{ in } \mathcal{S}'(\mathbb{R}^n)$$

with:

$$\sum |\lambda_k|^p \leq C \int_{\mathbb{R}^{n+1}_+} \left|\frac{\partial u}{\partial y}\right|^p y^\beta \, dx\, dy$$